Construction of quantum states by special superpositions of coherent states

G. Mogyorosi[1], P. Adam[1,2], E. Molnar[1], A. Varga[1], M. Mechler[3], and J. Janszky[1,2]

[1]Institute of Physics, University of Pécs, H-7624 Pécs, Ifjúság útja 6, Hungary

[2]Institute for Solid State Physics and Optics, Wigner Research Centre for Physics, HAS, H-1525 Budapest, P.O. Box 49, Hungary

[3]MTA-PTE High-Field Terahertz Research Group, H-7624, Pécs, Ifjúság útja 6, Hungary

Introduction

There has been a persistent interest in the construction and the production of nonclassical states of a harmonic oscillator system due to their potential applications in quantum optics and quantum information processing [1, 2]. Special attention has also been devoted to the idea of quantum state engineering, that is, to the preparation of arbitrary quantum states in the same experimental scheme [3]. An efficient method of quantum state engineering is to construct nonclassical states via discrete coherent-state superpositions. It has been shown that superpositions of even a small number of coherent states placed along a straight line or on a circle in phase space can approximate nonclassical field states with a high degree of accuracy [4, 5, 6]. This protocol exploits the effect of quantum interference between the constituent coherent states.

Inspired by these results, we consider quantum state engineering on an ellipse and on a specific 3×3 equidistant lattice of distance d in phase space. We show, by using an efficient numerical optimization method, that the superpositions of a small number of coherent states of these geometries can approximate certain nonclassical states with a high accuracy. We have optimized numerically the parameters of the chosen geometry and the coefficients of the coherent states via a genetic algorithm [7], in order to obtain the best feasible approximation. In this procedures we determined the misfit parameter [6]

$$\epsilon = 1 - |\langle \psi_N | \Psi \rangle|^2, \tag{1}$$

which minimizing by optimization. The quantity $|\langle \psi_N | \Psi \rangle|$ is the form of fidelity between the approximating coherent-state superposition $|\psi_N\rangle$ and the target quantum state $|\Psi\rangle$. Obviously, the described method for deriving the approximating coherent-state superpositions can only be used if a well-behaved one-dimensional coherent-state representation of the quantum state exists.

For certain states and parameter ranges the approximation is better than the corresponding one on a circle or along a line.

Coherent-state superpositions on an ellipse in phase space

Elliptical states can be defined as coherent-state superpositions on an ellipse in phase space

$$|\psi_N\rangle_{\text{ellipse}} = \mathcal{N} \sum_{k=1}^{N} c_k^{(\text{ellipse})} |\alpha_k\rangle, \tag{2}$$

where

$$\alpha_k = r_k \cdot e^{i\phi_k}, \quad \phi_k = \phi_0 + \frac{2\pi k}{N}, \qquad r_k = \left(\frac{\cos^2 \phi_k}{a^2} + \frac{\sin^2 \phi_k}{b^2} \right)^{\frac{1}{2}}, \tag{3}$$

where a and b are arbitrary real numbers. Here the constituent coherent states are placed equidistantly in their phase ϕ_k. Nonclassical properties of such states were analyzed only for states with constant coefficients $c_k = 1$ [8]. Our task is to find the set of coefficients $c_k^{(\text{ellipse})}$ in (2) and parameters a, b of the ellipse in (3), which minimize the misfit parameter $\epsilon^{(\text{ellipse})}$ in (1).

Coherent-state superpositions on a lattice in phase space

It has been shown that discrete coherent-state superpositions with variable coefficients on a lattice in phase space can be produced in traveling wave optics using only beam splitters and homodyne measurements [9, 10]. We consider the approximation of nonclassical states by such experimentally realizable superpositions on a lattice in phase space.

Let us consider the superposition of 9 coherent states

$$|\psi_9\rangle_{\text{lattice}} = \mathcal{N} \sum_{l=-1}^{1} \sum_{k=-1}^{1} c_{k,l}^{(\text{lattice})} |l \cdot d + k \cdot id\rangle, \tag{4}$$

on an equidistant lattice centered around the origin in phase space. In this equation $\mathcal{N}$ is a normalization constant and d is the distance between adjacent elements of the lattice. As in the case of the ellipse, the task is to find the optimal complex coefficients $c_{k,l}^{(\text{lattice})}$ and the distance d in (4). For the measure of the accuracy of the approximation to be optimized, we use the misfit parameter $\epsilon^{(\text{lattice})}$.

References

[1] Dodonov V V 2002 J. Opt. B: *Quantum Semiclass. Opt.* **4** R1-R33

[2] Dodonov V V, Man'ko M A, Man'ko V I and Vourdas A 2007 *J. Russ. Laser Res.* **28** 404

[3] Dell'Anno F, De Siena S and Illuminati F 2006 *Phys. Rep.* **428** 53-168

[4] Janszky J, Domokos P and Adam P 1993 *Phys. Rev.* A **48** 2213

[5] Janszky J, Domokos P, Szabó S and Adam P 1995 *Phys. Rev.* A **51** 4191

[6] Szabo S, Adam P, Janszky J and Domokos P 1996 *Phys. Rev.* A **53** 2698

[7] Goldberg D. E. 1989 *Genetic Algorithms in Search, Optimization and Machine Learning* (Boston: Addison-Wesley Publishing Co., Inc.)

[8] Wang Y, Liao Q, Liu Z, Wang J and Liu S 2011 *Opt. Commun.* **284** 282

[9] Adam P, Kiss T, Mechler M, Darazs Z 2010 *Phys. Scr.* **T140** 014011

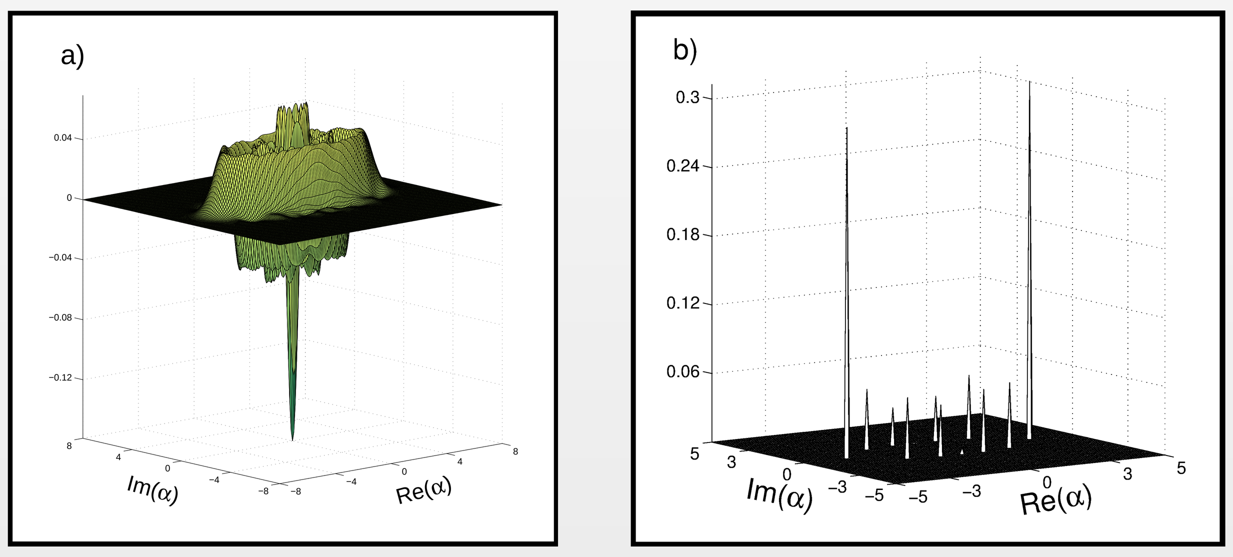
[10] Molnar E, Varga A, Mogyorosi G and Adam P 2014 *21st Central European Workshop on Quantum Optics (Brussels)* (book of abstracts) p 213

Results

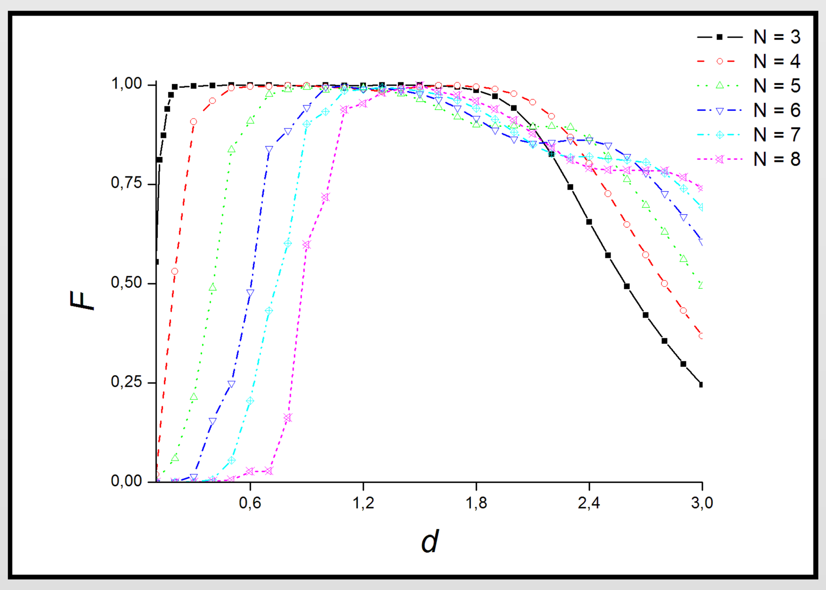
- The minimal misfits $\epsilon^{(\text{ellipse})}$ and $\epsilon^{(\text{line})}$ of the approximation of the squeezed number states on an ellipse and along a line with 12 coherent states and the optimal parameters a_{opt}, b_{opt} of the ellipse and d_{opt} of the line for various photon numbers and for a constant squeezing parameter $\zeta = 0.5$.

State	$\epsilon^{(\text{ellipse})}$	a_{opt}	b_{opt}	$\epsilon^{(\text{line})}$	d_{opt}
$\lvert 0, 0.5, 0\rangle$	6×10^{-4}	1.25	0.17	1.4×10^{-5}	0.27
$\lvert 3, 0.5, 0\rangle$	0.001	2.63	0.97	0.0015	0.47
$\lvert 5, 0.5, 0\rangle$	0.006	3.39	1.44	0.018	0.6
$\lvert 7, 0.5, 0\rangle$	0.0172	4.09	1.54	0.052	0.74

Approximating squeezed number state $|5, 0.5, 0\rangle$ by $N = 12$ coherent states on an ellipse (a) Wigner function $W(\alpha)$ of the approximating coherent-state superposition, (b) pins showing the positions and the absolute values of the coefficients of the constituent coherent states.



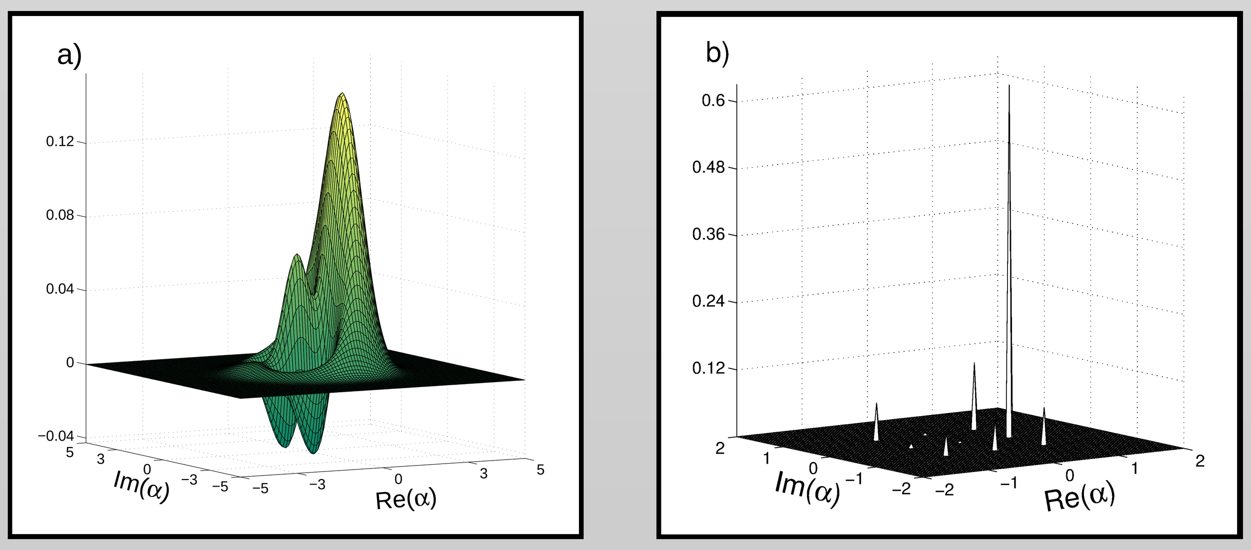
- For the number states we have found that high accuracy can be achieved not only at the optimal value of d but in a range of distances around this value. The figure shows that as the photon number n increases, the width of the acceptable range of high accuracy decreases. The lower limit of this range shifts to the direction of larger distances while the upper limit shows a more complicated behavior, but the accuracy of the approximation relevantly decreases for distances $d > 1.7$.



- The misfits $\epsilon^{(\text{lattice})}$ and $\epsilon^{(\text{circle})}$ of the approximation of number-state superpositions on a lattice and on a circle with 9 coherent states and the optimal distances d_{opt} and radii R_{opt} for increasing number of superposed states.

State	$\epsilon^{(\text{lattice})}$	d_{opt}	$\epsilon^{(\text{circle})}$	R_{opt}
$\frac{1}{\sqrt{2}}(\lvert 0\rangle + \lvert 1\rangle)$	2×10^{-6}	0.257	2.2×10^{-5}	0.54
$\frac{1}{\sqrt{3}}(\lvert 0\rangle + \lvert 1\rangle + \lvert 2\rangle)$	1.8×10^{-5}	0.53	4×10^{-4}	0.9
$\frac{1}{2}(\lvert 0\rangle + \lvert 1\rangle + \lvert 2\rangle + \lvert 3\rangle)$	6.6×10^{-5}	0.79	0.0016	1.17
$\frac{1}{\sqrt{5}}(\lvert 0\rangle + \lvert 1\rangle + \lvert 2\rangle + \lvert 3\rangle + \lvert 4\rangle)$	0.0017	1.37	0.0061	1.32
$\frac{1}{\sqrt{72}}(7\lvert 0\rangle + 3\lvert 1\rangle + 2\lvert 2\rangle + \lvert 3\rangle + 3\lvert 4\rangle)$	0.0016	1.33	0.0035	1.15

Approximating number state superposition $\frac{1}{\sqrt{3}}(|0\rangle + |1\rangle + |2\rangle)$ by $N = 9$ coherent states on a lattice (a) Wigner function $W(\alpha)$ of the approximating coherentstate superposition, (b) pins showing the positions and the absolute values of the coefficients of the constituent coherent states.



Conclusion

We have considered the optimal approximation of certain quantum states with the superposition of a finite number of coherent states on an ellipse and on a lattice in phase space. All the coherent-state superpositions we have considered are feasible in current experiments. We have optimized numerically the parameters of the chosen geometry and the coefficients of the coherent states via a genetic algorithm, in order to obtain the best feasible approximation.

First, we placed the coherent states equidistantly in their phase on an ellipse. For squeezed number states in a certain parameter range, the elliptical approximation outperforms the one on a line.

Next we have considered a 3×3 equidistant lattice around the origin of phase space. A relatively large set of states, including special numberstate superpositions, appear to be better approximated in this geometry than with a superposition on a circle.

Acknowledgement

The financial support by SROP-4.1.1.C-12/1/KONV-2012-0005 and by the Hungarian Scientific Research Fund OTKA (Contract No. K83858) is acknowledged.